

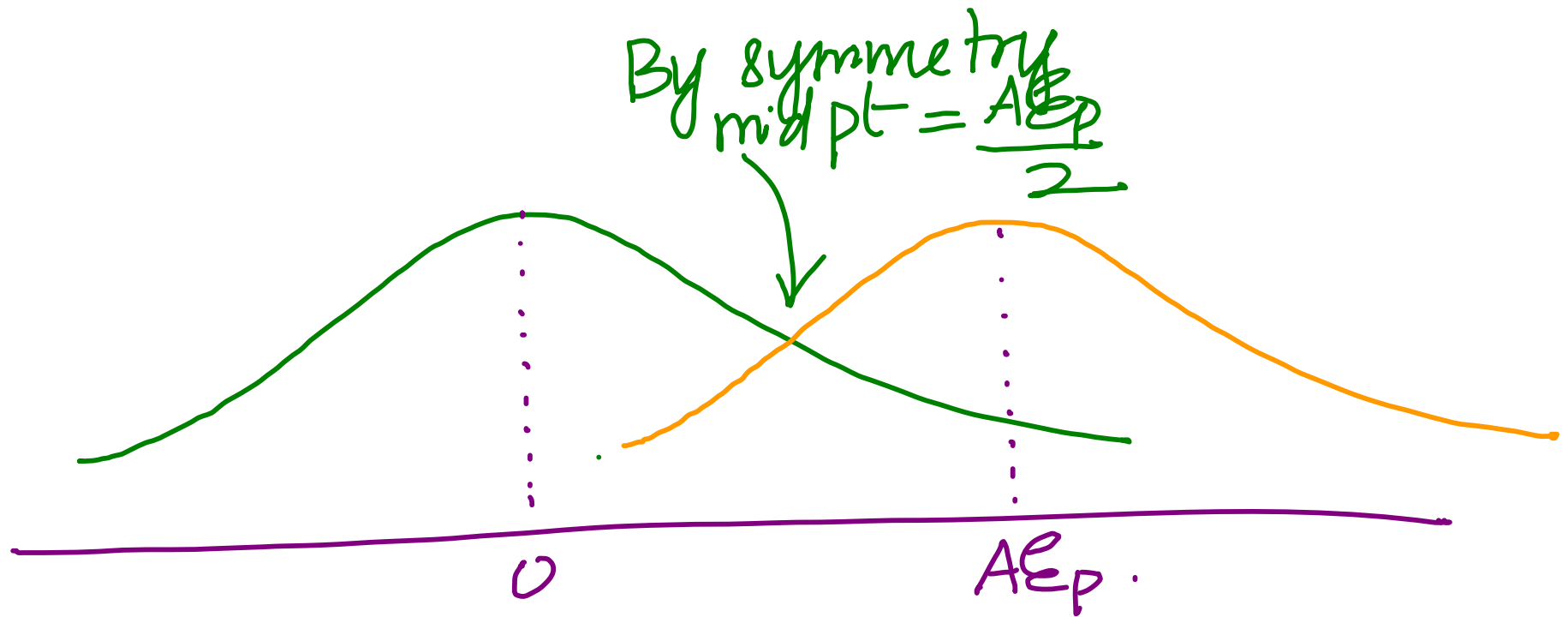
$$\Gamma(\pi) = \begin{cases} A \epsilon_p + \tilde{\eta} \\ \tilde{\eta} \end{cases}$$

Gaussian
mean $\frac{N_0}{2} \epsilon_p$ -
if $a_0 = A$

if $a_0 = A$

$$\checkmark N(A \epsilon_p, \frac{N_0}{2} \epsilon_p)$$

$$\checkmark N(0, \frac{N_0}{2} \epsilon_p)$$



Therefore/

Decide $a_0 = A$

if $r(T) \geq \frac{A\epsilon_p}{2}$

Decide $a_0 = 0$

if $r(T) < \frac{A\epsilon_p}{2}$

$$r(T) = \begin{cases} A\epsilon_p + \tilde{n} & a_0 = A \\ \tilde{n} & a_0 = 0 \end{cases}$$

$$\tilde{r}(T) = r(T) - \frac{A\epsilon_p}{2}$$

$$\tilde{r}(T) = \begin{cases} \frac{A\epsilon_p}{2} + \tilde{n} & \text{if } a_0 = A \\ -\frac{A\epsilon_p}{2} + \tilde{n} & \text{if } a_0 = 0. \end{cases}$$

Similar to BPSK with $A\epsilon_p$.
replaced by $A\epsilon_p/2$

Decide $a_0 = A$

if $\tilde{r}(T) \geq 0$

$$\Rightarrow r(T) - \frac{A\epsilon_p}{2} \geq 0$$

$$\Rightarrow \boxed{r(T) \geq \frac{A\epsilon_p}{2}}$$

$r(T) \geq \frac{A\epsilon_p}{2} \Rightarrow \text{Decide } a_0 = A$

$r(T) < \frac{A\epsilon_p}{2} \Rightarrow \text{Decide } a_0 = -A.$

$$\tilde{r}(T) = \begin{cases} \frac{AE_p}{2} + \tilde{n} \\ -\frac{AE_p}{2} + \tilde{n} \end{cases}$$

Bit Error Rate or
Probability of bit error
similar to BPSK with AE_p
replaced by $\frac{AE_p}{2}$.

$$P_e = Q\left(\frac{AE_p/2}{\sqrt{\frac{N_0}{2}E_p}}\right)$$

$$= Q \left(\sqrt{\frac{A^2 E_p}{2 N_0}} \right)$$

$$E_p = 1 \quad A^2 = 2 E_p.$$

$$P_e = Q \left(\sqrt{\frac{2 E_b}{2 N_0}} \right)$$

$P_e = Q \left(\sqrt{\frac{E_b}{N_0}} \right)$

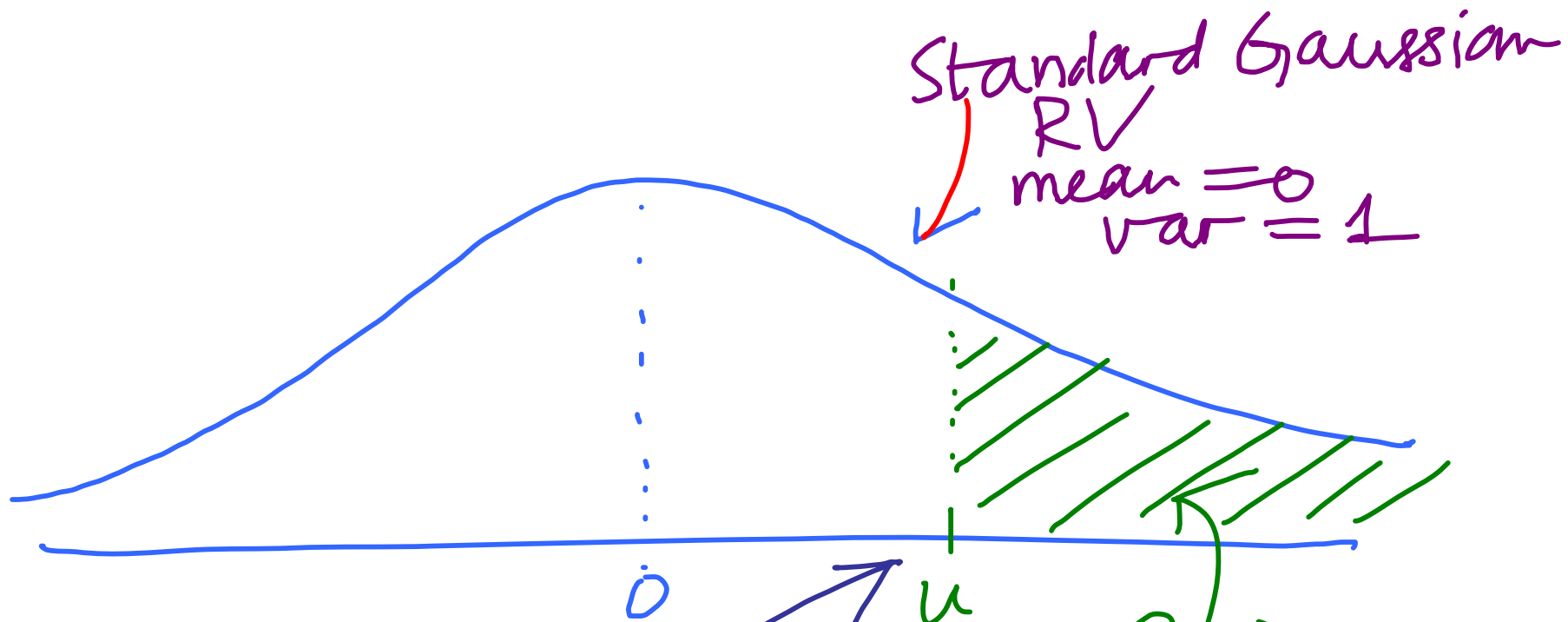
Probability of Error
for ASK.

Compare this with prob of error of BPSK

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_{e,ASK} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$



This is a decreasing
function of u .

$Q(u)$
= Prob that
Standard Gaussian
RV $\geq u$.

$$= \int_u^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

$$Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \leq Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$



$$P_{e,BPSK} \leq P_{e,ASK}$$

Both modulation schemes have same bit Energy = E_b .

Fair

How much improvement in BER does BPSK provide?

$$Q\left(\sqrt{\frac{2 \cdot \frac{1}{2} E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BER of BPSK for $\frac{1}{2} E_b$.

For same BER, BPSK needs $\frac{1}{2}$ the average bit energy as that of ASK.

$$\Rightarrow E_{b, \text{BPSK}} = \frac{1}{2} E_{b, \text{ASK}}$$

$$\begin{aligned} \Rightarrow 10 \log_{10} E_{b, \text{BPSK}} \\ = 10 \log_{10} \frac{1}{2} \\ + 10 \log_{10} E_{b, \text{ASK}}. \end{aligned}$$

$$\begin{aligned} \Rightarrow 10 \log_{10} E_{b, \text{BPSK}} - 10 \log_{10} E_{b, \text{ASK}} \\ = \frac{-10 \log_{10} 2}{\approx 3} = -3 \text{ dB} \end{aligned}$$

$$E_{b,BPSK} (dB) - E_{b,ASK} (dB) = -3 dB$$

For same BER, BPSK requires 3dB lower average energy/bit E_b .
or basically ASK requires 3dB higher E_b when compared to BPSK for the same BER.